SOLUTION OF EXERCISE # 2.1

Exercise # 2.1

Q.1: Expand the following by the binomial formula.

(i) $\left(\frac{x}{2}-\frac{2}{y}\right)^4$

Sol. $\left(\frac{x}{2}-\frac{2}{y}\right)^4$

$$=\binom{4}{0}\left(\frac{x}{2}\right)^4\left(\frac{2}{y}\right)^0-\binom{4}{1}\left(\frac{x}{2}\right)^3\left(\frac{2}{y}\right)^1+\binom{4}{2}\left(\frac{x}{2}\right)^2\left(\frac{2}{y}\right)^2-\binom{4}{3}\left(\frac{x}{2}\right)^1\left(\frac{2}{y}\right)^3+\binom{4}{4}\left(\frac{x}{2}\right)^0\left(\frac{2}{y}\right)^4$$

$$=1\left(\frac{x^4}{16}\right)(1)-4\left(\frac{x^3}{8}\right)\left(\frac{2}{y}\right)+6\left(\frac{x^2}{4}\right)\left(\frac{4}{y^2}\right)-4\left(\frac{x}{2}\right)\left(\frac{8}{y^3}\right)+1(1)\left(\frac{16}{y^4}\right)$$

$$=\boxed{\frac{x^4}{16}-\frac{x^3}{y}+6\frac{x^2}{y^2}-16\frac{x}{y^3}+\frac{16}{y^4}}$$

(ii) $\left(\frac{2x}{3}-\frac{3}{2x}\right)^5$

Sol. $\left(\frac{2x}{3}-\frac{3}{2x}\right)^5$

$$=\binom{5}{0}\left(\frac{2x}{3}\right)^5\left(\frac{3}{2x}\right)^0-\binom{5}{1}\left(\frac{2x}{3}\right)^4\left(\frac{3}{2x}\right)^1+\binom{5}{2}\left(\frac{2x}{3}\right)^3\left(\frac{3}{2x}\right)^2$$
$$-\binom{5}{3}\left(\frac{2x}{3}\right)^2\left(\frac{3}{2x}\right)^3+\binom{5}{4}\left(\frac{2x}{3}\right)^1\left(\frac{3}{2x}\right)^4-\binom{5}{5}\left(\frac{2x}{3}\right)^0\left(\frac{3}{2x}\right)^5$$

$$=1\left(\frac{32x^5}{243}\right)(1)-5\left(\frac{16x^4}{81}\right)\left(\frac{3}{2x}\right)+10\left(\frac{8x^3}{27}\right)\left(\frac{9}{4x^2}\right)$$
$$-10\left(\frac{4x^2}{9}\right)\left(\frac{27}{8x^3}\right)+5\left(\frac{2x}{3}\right)\left(\frac{81}{16x^4}\right)-1(1)\left(\frac{243}{32x^5}\right)$$

$$=\boxed{\frac{32x^5}{243}-\frac{40x^3}{27}+\frac{20x}{3}-\frac{15}{x}+\frac{135}{8x^3}-\frac{243}{32x^5}}$$

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(iii) $\left(x+\frac{1}{x}\right)^4$

Sol. $\left(x+\frac{1}{x}\right)^4$

$$=\binom{4}{0}(x)^4\left(\frac{1}{x}\right)^0+\binom{4}{1}(x)^3\left(\frac{1}{x}\right)^1+\binom{4}{2}(x)^2\left(\frac{1}{x}\right)^2+\binom{4}{3}(x)^1\left(\frac{1}{x}\right)^3+\binom{4}{4}(x)^0\left(\frac{1}{x}\right)^4$$

$$=(1)x^4(1)+4x^3\left(\frac{1}{x}\right)+6x^2\left(\frac{1}{x^2}\right)+4x\left(\frac{1}{x^3}\right)+(1)(1)\left(\frac{1}{x^4}\right)$$

$$=\boxed{x^4+4x^2+6+\frac{4}{x^2}+\frac{1}{x^4}}$$

(iv) $(2x-y)^5$

Sol. $(2x-y)^5$

$$=\binom{5}{0}(2x)^5y^0-\binom{5}{1}(2x)^4y^1+\binom{5}{2}(2x)^3y^2-\binom{5}{3}(2x)^2y^3+\binom{5}{4}(2x)^1y^4-\binom{5}{5}(2x)^0y^5$$

$$=1(32x^5)(1)-5(16x^4)y+10(8x^3)y^2-10(4x^2)y^3+5(2x)y^4-1(1)y^5$$

$$=\boxed{32x^5-80x^4y+80x^3y^2-40x^2y^3+10xy^4-y^5}$$

(v) $\left(2a-\frac{x}{a}\right)^7$

Sol. $\left(2a-\frac{x}{a}\right)^7$

$$=\binom{7}{0}(2a)^7\left(\frac{x}{a}\right)^0-\binom{7}{1}(2a)^6\left(\frac{x}{a}\right)^1+\binom{7}{2}(2a)^5\left(\frac{x}{a}\right)^2-\binom{7}{3}(2a)^4\left(\frac{x}{a}\right)^3$$

$$+\binom{7}{4}(2a)^3\left(\frac{x}{a}\right)^4-\binom{7}{5}(2a)^2\left(\frac{x}{a}\right)^5+\binom{7}{6}(2a)^1\left(\frac{x}{a}\right)^6-\binom{7}{7}(2a)^0\left(\frac{x}{a}\right)^7$$

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$$=1\left(128a^7\right)(1)-7\left(64a^6\right)\left(\frac{x}{a}\right)+21\left(32a^5\right)\left(\frac{x^2}{a^2}\right)-35\left(16a^4\right)\left(\frac{x^3}{a^3}\right)$$

$$+35\left(8a^3\right)\left(\frac{x^4}{a^4}\right)-21\left(4a^2\right)\left(\frac{x^5}{a^5}\right)+7(2a)\left(\frac{x^6}{a^6}\right)-1(1)\left(\frac{x^7}{a^7}\right)$$

$$=\boxed{128a^7-448a^5x+672a^3x^2-560ax^3+280\frac{x^4}{a}-84\frac{x^5}{a^3}+14\frac{x^6}{a^5}-\frac{x^7}{a^7}}$$

(vi) $\left(\frac{x}{y}-\frac{y}{x}\right)^4$

Sol. $\left(\frac{x}{y}-\frac{y}{x}\right)^4$

$$=\binom{4}{0}\left(\frac{x}{y}\right)^4\left(\frac{y}{x}\right)^0-\binom{4}{1}\left(\frac{x}{y}\right)^3\left(\frac{y}{x}\right)^1+\binom{4}{2}\left(\frac{x}{y}\right)^2\left(\frac{y}{x}\right)^2-\binom{4}{3}\left(\frac{x}{y}\right)^1\left(\frac{y}{x}\right)^3+\binom{4}{4}\left(\frac{x}{y}\right)^0\left(\frac{y}{x}\right)^4$$

$$=1\left(\frac{x^4}{y^4}\right)(1)-4\left(\frac{x^3}{y^3}\right)\left(\frac{y}{x}\right)+6\left(\frac{x^2}{y^2}\right)\left(\frac{y^2}{x^2}\right)-4\left(\frac{x}{y}\right)\left(\frac{y^3}{x^3}\right)+1(1)\left(\frac{y^4}{x^4}\right)$$

$$=\boxed{\frac{x^4}{y^4}-4\frac{x^2}{y^2}+6-4\frac{y^2}{x^2}+\frac{y^4}{x^4}}$$

Q.2: Compute to two decimal places of decimal by use of binomial formula.

(i) $(1.02)^4$ (IIA-2017)

Sol. $(1.02)^4=(1+0.02)^4$

$$=\binom{4}{0}(1)^4(0.02)^0+\binom{4}{1}(1)^3(0.02)^1+\binom{4}{2}(1)^2(0.02)^2+\ldots$$

$$=(1)(1)(1)+4(1)(0.02)+6(1)(0.0004)+\ldots$$

$$=1+0.08+0.0024+\ldots=1.0824=\boxed{1.08}$$

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(ii) $(0.98)^6$ (IA-2018)

Sol. $(0.98)^6 = (1-0.02)^6$

$$= \binom{6}{0}(1)^6(0.02)^0 - \binom{6}{1}(1)^5(0.02)^1 + \binom{6}{2}(1)^4(0.02)^2 + \ldots$$

$$= (1)(1)(1) - 6(1)(0.02) + 15(1)(0.0004) + \ldots$$

$$= 1 - 0.12 + 0.006 + \ldots = 0.886 = \boxed{0.89}$$

(iii) $(2.03)^5$ (IIA-2015)

Sol. $(2.03)^5 = (2+0.03)^5$

$$= \binom{5}{0}(2)^5(0.03)^0 + \binom{5}{1}(2)^4(0.03)^1 + \binom{5}{2}(2)^3(0.03)^2 + \ldots$$

$$= (1)(32)(1) + 5(16)(0.03) + 10(8)(0.0009) + \ldots$$

$$= 32 + 2.4 + 0.072 + \ldots = 34.472 = \boxed{34.47}$$

Q.3: Find the value of:

(i) $(x+y)^5 + (x-y)^5$ (IIA-2021)

Sol. $(x+y)^5 + (x-y)^5$

$$= \left(\binom{5}{0}x^5y^0 + \binom{5}{1}x^4y^1 + \binom{5}{2}x^3y^2 + \binom{5}{3}x^2y^3 + \binom{5}{4}x^1y^4 + \binom{5}{5}x^0y^5\right)$$

$$+ \left(\binom{5}{0}x^5y^0 - \binom{5}{1}x^4y^1 + \binom{5}{2}x^3y^2 - \binom{5}{3}x^2y^3 + \binom{5}{4}x^1y^4 - \binom{5}{5}x^0y^5\right)$$

$$= x^5 + \cancel{5x^4y} + 10x^3y^2 + \cancel{10x^2y^3} + 5xy^4 + \cancel{y^5} + x^5 - \cancel{5x^4y} + 10x^3y^2$$

$$- \cancel{10x^2y^3} + 5xy^4 - \cancel{y^5} = \boxed{2x^5 + 20x^3y^2 + 10xy^4}$$

(ii) $\left(x+\sqrt{2}\right)^4 + \left(x-\sqrt{2}\right)^4$

Sol. $\left(x+\sqrt{2}\right)^4 + \left(x-\sqrt{2}\right)^4$

SOLUTION OF EXERCISE # 2.1

$$=\left(\binom{4}{0}x^4(\sqrt{2})^0+\cancel{\binom{4}{1}x^3(\sqrt{2})^1}+\binom{4}{2}x^2(\sqrt{2})^2+\cancel{\binom{4}{3}x^1(\sqrt{2})^3}+\binom{4}{4}x^0(\sqrt{2})^4\right)$$

$$+\left(\binom{4}{0}x^4(\sqrt{2})^0-\cancel{\binom{4}{1}x^3(\sqrt{2})^1}+\binom{4}{2}x^2(\sqrt{2})^2-\cancel{\binom{4}{3}x^1(\sqrt{2})^3}+\binom{4}{4}x^0(\sqrt{2})^4\right)$$

$$=(1)x^4(1)+6x^2(2)+(1)(1)(4)+(1)x^4(1)+6x^2(2)+(1)(1)(4)$$

$$=x^4+12x^2+4+x^4+12x^2+4=\boxed{2x^4+24x^2+8}$$

Q.4: Expanding the following in ascending power of 'x'.

(i) $(1-x+x^2)^4$

Sol. $(1-x+x^2)^4=\left[(1-x)+x^2\right]^4$

$$=\binom{4}{0}(1-x)^4(x^2)^0+\binom{4}{1}(1-x)^3(x^2)^1+\binom{4}{2}(1-x)^2(x^2)^2+\binom{4}{3}(1-x)^1(x^2)^3+\binom{4}{4}(1-x)^0(x^2)^4$$

$$=1(1-x)^4(1)+4(1-x)^3(x^2)+6(1-x)^2(x^4)+4(1-x)(x^6)+1(1)(x^8)$$

$$=(1-x)^4+4x^2(1-x)^3+6x^4(1-x)^2+4x^6(1-x)+x^8$$

$$=\left[\binom{4}{0}(1)^4x^0-\binom{4}{1}(1)^3x^1+\binom{4}{2}(1)^2x^2-\binom{4}{3}(1)^1x^3+\binom{4}{4}(1)^0x^4\right]$$

$$+4x^2\left[\binom{3}{0}(1)^3x^0-\binom{3}{1}(1)^2x^1+\binom{3}{2}(1)^1x^2-\binom{3}{3}(1)^0x^3\right]$$

$$+6x^4(1-2x+x^2)+4x^6(1-x)+x^8$$

$$=(1-4x+6x^2-4x^3+x^4)+4x^2(1-3x+3x^2-x^3)$$

$$+6x^4-12x^5+6x^6+4x^6-4x^7+x^8$$

$$=1-4x+6x^2-4x^3+x^4+4x^2-12x^3+12x^4-4x^5$$

$$+6x^4-12x^5+6x^6+4x^6-4x^7+x^8$$

$$=\boxed{1-4x+10x^2-16x^3+19x^4-16x^5+10x^6-4x^7+x^8}$$

(ii) $(2+x-x^2)^4$

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Sol. $(2+x-x^2)^4 = \left[(2+x)-x^2\right]^4$

$$=\binom{4}{0}(2+x)^4(x^2)^0-\binom{4}{1}(2+x)^3(x^2)^1+\binom{4}{2}(2+x)^2(x^2)^2-\binom{4}{3}(2+x)^1(x^2)^3+\binom{4}{4}(2+x)^0(x^2)^4$$

$$=(1)(2+x)^4(1)-(4)(2+x)^3(x^2)+(6)(2+x)^2(x^4)-(4)(2+x)^1(x^6)+(1)(1)(x^8)$$

$$=(2+x)^4-4x^2(2+x)^3+6x^4(2+x)^2-4x^6(2+x)+x^8$$

$$=\left[\binom{4}{0}(2)^4x^0+\binom{4}{1}(2)^3x^1+\binom{4}{2}(2)^2x^2+\binom{4}{3}(2)^1x^3+\binom{4}{4}(2)^0x^4\right]$$

$$-4x^2\left[\binom{3}{0}(2)^3x^0+\binom{3}{1}(2)^2x^1+\binom{3}{2}(2)^1x^2+\binom{3}{3}(2)^0x^3\right]$$

$$+6x^4\left(2^2+x^2+2(2)(x)\right)-4x^6(2+x)+x^8$$

$$=\left[(1)(16)(1)+4(8)x+6(4)x^2+4(2)x^3+(1)(1)(1)\right]$$

$$-4x^2\left[(1)(8)(1)+3(4)x+3(2)x^2+1(1)x^3\right]$$

$$+6x^4\left(4+x^2+4x\right)-4x^6(2+x)+x^8$$

$$=\left[16+32x+24x^2+8x^3+1\right]-4x^2\left[8+12x+6x^2+x^3\right]$$

$$+6x^4\left(4+x^2+4x\right)-4x^6(2+x)+x^8$$

$$=16+32x+24x^2+8x^3+1-32x^2-48x^3-24x^4-4x^5$$

$$+24x^4+6x^6+24x^5-8x^6-4x^7+x^8$$

$$=\boxed{16+32x-8x^2-40x^3+x^4+20x^5-2x^6-4x^7+x^8}$$

Q.5: Find:

(i) The 5th term in the expansion of $\left(2x^2-\frac{3}{x}\right)^{10}$ (IA-2019)

Sol. Here $n=10$, $a=2x^2$, $b=-\frac{3}{x}$, $r=4$

$$T_{r+1}=\binom{n}{r}a^{n-r}b^r$$

SOLUTION OF EXERCISE # 2.1

$$T_{4+1}=\binom{10}{4}\left(2x^2\right)^{10-4}\left(-\frac{3}{x}\right)$$

$$T_5=210\left(2x^2\right)^6\left(-\frac{3}{x}\right)^4=210\left(64x^{12}\right)\left(\frac{81}{x^4}\right)=\boxed{1088640x^8}$$

(ii) **The 6th term in the expansion of** $\left(x^2+\frac{y}{2}\right)^{15}$

Sol. Here $n=15$, $a=x^2$, $b=\frac{y}{2}$, $r=5$

$$T_{r+1}=\binom{n}{r}a^{n-r}b^r$$

$$T_{5+1}=\binom{15}{5}(x^2)^{15-5}\left(\frac{y}{2}\right)^5$$

$$T_6=3003(x^2)^{10}\left(\frac{y^5}{32}\right)=\boxed{\frac{3003}{32}x^{20}y^5}$$

(iii) **The 8th term in the expansion of** $\left(\sqrt{x}+\frac{2}{\sqrt{x}}\right)^{12}$

Sol. Here $n=12$, $a=\sqrt{x}$, $b=\frac{2}{\sqrt{x}}$, $r=7$

$$T_{r+1}=\binom{n}{r}a^{n-r}b^r \Rightarrow T_{7+1}=\binom{12}{7}\left(\sqrt{x}\right)^{12-7}\left(\frac{2}{\sqrt{x}}\right)^7$$

$$T_8=792\left(x^{1/2}\right)^5\left(\frac{128}{x^{\frac{7}{2}}}\right)=101376\frac{x^{5/2}}{x^{7/2}}$$

$$T_8=\frac{101376}{x^{\frac{7}{2}-\frac{5}{2}}}=\frac{101376}{x^{\frac{7-5}{2}}}=\frac{101376}{x^{\frac{2}{2}}}=\boxed{\frac{101376}{x}}$$

SOLUTION OF EXERCISE # 2.1

(iv) **The 7th term in the expansion of** $\left(\frac{4x}{5}-\frac{5}{2x}\right)^9$

Sol. Here $n = 9$, $a=\frac{4x}{5}$, $b=\frac{-5}{2x}$, $r = 6$

$$T_{r+1}=\binom{n}{r}a^{n-r}b^r$$

$$T_{6+1}=\binom{9}{6}\left(\frac{4x}{5}\right)^{9-6}\left(\frac{-5}{2x}\right)^6$$

$$T_7=84\left(\frac{4x}{5}\right)^3\left(\frac{-5}{2x}\right)^6$$

$$T_7=84\left(\frac{64x^3}{125}\right)\left(\frac{15625}{64x^6}\right)=\frac{10500}{x^{6-3}}=\boxed{\frac{10500}{x^3}}$$

Q.6: Find the middle term/terms of the following expansions:

(i) $\left(3x^2+\frac{1}{2x}\right)^{10}$ (IIA-2017)

Sol. Here $n = 10$, $a = 3x^2$, $b=\frac{1}{2x}$,

As $n = 10$ (Even), so

$$\text{Middle term} = \left(\frac{n}{2}+1\right)^{th}\text{ term}$$

$$=\left(\frac{10}{2}+1\right)^{th}\text{ term} = (5+1)^{th}\text{ term} = 6^{th}\text{ term}$$

For 6^{th} term, put $r = 5$

$$T_{r+1}=\binom{n}{r}a^{n-r}b^r$$

SOLUTION OF EXERCISE # 2.1

$$T_{5+1} = \binom{10}{5}(3x^2)^{10-5}\left(\frac{1}{2x}\right)^5 = 252(3x^2)^5\left(\frac{1}{2x}\right)^5$$

$$T_6 = 252(243x^{10})\left(\frac{1}{32x^5}\right)$$

$$T_6 = \frac{61236}{32}x^{10-5} = \boxed{\frac{15309}{8}x^5}$$

(ii) $\left(\frac{a}{2} - \frac{b}{3}\right)^{11}$ (IA-2021)

Sol. Here $n = 11$, $a = \frac{a}{2}$, $b = -\frac{b}{3}$

As $n = 11$ (Odd), so

$$\text{Middle terms} = \left(\frac{n+1}{2}\right)^{th} + \left(\frac{n+3}{2}\right)^{th} \text{ terms.}$$

$$= \left(\frac{11+1}{2}\right)^{th} + \left(\frac{11+3}{2}\right)^{th} \text{ terms.}$$

$$= 6^{th} + 7^{th} \text{ terms}$$

For 6^{th} term, put $r = 5$

$$T_{r+1} = \binom{n}{r}a^{n-r}b^r$$

$$T_{5+1} = \binom{11}{5}\left(\frac{a}{2}\right)^{11-5}\left(-\frac{b}{3}\right)^5$$

$$T_6 = 462\left(\frac{a^6}{64}\right)\left(-\frac{b^5}{243}\right)$$

$$T_6 = -\frac{462a^6b^5}{15552}$$

$$T_6 = \boxed{-\frac{77a^6b^5}{2592}}$$

For 7^{th} term, put $r = 6$

$$T_{r+1} = \binom{n}{r}a^{n-r}b^r$$

$$T_{6+1} = \binom{11}{6}\left(\frac{a}{2}\right)^{11-6}\left(-\frac{b}{3}\right)^6$$

$$T_7 = 462\left(\frac{a^5}{32}\right)\left(\frac{b^6}{729}\right)$$

$$T_7 = \frac{462a^5b^6}{23328}$$

$$T_7 = \boxed{\frac{77a^5b^6}{3888}}$$

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SOLUTION OF EXERCISE # 2.1

(iii) $\left(2x+\frac{1}{x}\right)^7$

Sol. Here $n = 7$, $a = 2x$, $b = \frac{1}{x}$

As $n = 7$ (odd)

So Middle terms $= \left(\frac{n+1}{2}\right)^{th} + \left(\frac{n+3}{2}\right)^{th}$ terms.

$= \left(\frac{7+1}{2}\right)^{th} + \left(\frac{7+3}{2}\right)^{th}$ terms.

$= 4^{th} + 5^{th}$ terms

For 4^{th} term, put $r = 3$

$$T_{r+1} = \binom{n}{r} a^{n-r} b^r$$

$$T_{3+1} = \binom{7}{3} (2x)^{7-3} \left(\frac{1}{x}\right)^3$$

$$T_4 = 35(16x^4)\left(\frac{1}{x^3}\right)$$

$$\boxed{T_4 = 560x}$$

For 5^{th} term, put $r = 4$

$$T_{r+1} = \binom{n}{r} a^{n-r} b^r$$

$$T_{4+1} = \binom{7}{4} (2x)^{7-4} \left(\frac{1}{x}\right)^4$$

$$T_5 = 35(8x^3)\left(\frac{1}{x^4}\right)$$

$$\boxed{T_5 = \frac{280}{x}}$$

Q.7: Find the specified term in the expansion of:

(i) $\left(2x^2-\frac{3}{x}\right)^{10}$: **term involving x^5**

Sol. Here $n = 10$, $a = 2x^2$, $b = -\frac{3}{x}$, $r = ?$ **(IIA-2019)**

$$T_{r+1} = \binom{n}{r} a^{n-r} b^r$$

$$T_{r+1} = \binom{10}{r} (2x^2)^{10-r} \left(-\frac{3}{x}\right)^r$$

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$$T_{r+1} = \binom{10}{r}(2)^{10-r}x^{20-2r}\frac{(-3)^r}{x^r}$$

$$T_{r+1} = \binom{10}{r}(2)^{10-r}(-3)^r x^{20-2r-r}$$

$$T_{r+1} = \binom{10}{r}(2)^{10-r}(-3)^r x^{20-3r} \rightarrow (i)$$

Put

$20 - 3r = 5$

$-3r = 5 - 20$

$= -15$

$r = \frac{-15}{-3}$

$r = \boxed{5}$

Put $r = 5$ in eq.(i)

$$T_{5+1} = \binom{10}{5}(2)^{10-5}(-3)^5 x^{20-3(5)}$$

$$T_6 = 252(2)^5(-3)^5 x^{20-15}$$

$$T_6 = 252(32)(-243)x^5$$

$$T_6 = \boxed{-1959552x^5}$$

(ii) $\left(2x^2 - \frac{1}{2x}\right)^{10}$: **term involving x^5**

Sol. Here $n = 10$, $a = 2x^2$, $b = \frac{-1}{2x}$, $r = ?$

$$T_{r+1} = \binom{n}{r}a^{n-r}b^r$$

$$T_{r+1} = \binom{10}{r}(2x^2)^{10-r}\left(-\frac{1}{2x}\right)^r$$

$$T_{r+1} = \binom{10}{r}(2)^{10-r}x^{20-2r}\frac{(-1)^r}{2^r x^r}$$

$$T_{r+1} = \binom{10}{r}(2)^{10-r-r}(-1)^r x^{20-2r-r}$$

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SOLUTION OF EXERCISE # 2.1

$$T_{r+1} = \binom{10}{r}(2)^{10-2r}(-1)^r x^{20-3r} \rightarrow (i)$$

Put

$20 - 3r = 5$

$-3r = 5 - 20$

$-3r = 5 - 20$

$-3r = -15$

$r = \dfrac{-15}{-3} = \boxed{5}$

Put $r = 5$ in eq.(i)

$T_{5+1} = \binom{10}{5}(2)^{10-2(5)}(-1)^5 x^{20-3(5)}$

$T_6 = 252(2)^0(-1)x^{20-15}$

$T_6 = 252(1)(-1)x^5$

$T_6 = \boxed{-252x^5}$

(iii) $\left(x^3 + \dfrac{1}{x}\right)^7$: **term involving x^9**

Sol. Here $n = 7$, $a = x^3$, $b = \dfrac{1}{x}$, $r = ?$

$$T_{r+1} = \binom{n}{r}a^{n-r}b^r$$

$$T_{r+1} = \binom{7}{r}(x^3)^{7-r}\left(\frac{1}{x}\right)^r = \binom{7}{r}x^{21-3r}\frac{1}{x^r}$$

$$T_{r+1} = \binom{7}{r}x^{21-3r-r}$$

$$T_{r+1} = \binom{7}{r}x^{21-4r} \rightarrow (i)$$

Put $21 - 4r = 9$

$-4r = 9 - 21$

$-4r = -12$

$r = \dfrac{-12}{-4} = \boxed{3}$

Put $r = 3$ in eq.(i)

$T_{3+1} = \binom{7}{3}x^{21-4(3)}$

$T_4 = \boxed{35x^9}$

SOLUTION OF EXERCISE # 2.1

(iv) $\left(\frac{x}{2}-\frac{4}{x}\right)^8$: term involving x^2

Sol. Here $n = 8$, $a = \frac{x}{2}$, $b = -\frac{4}{x}$, $r = ?$

$$T_{r+1} = \binom{n}{r} a^{n-r} b^r$$

$$T_{r+1} = \binom{8}{r}\left(\frac{x}{2}\right)^{8-r}\left(-\frac{4}{x}\right)^r$$

$$T_{r+1} = \binom{8}{r}\frac{x^{8-r}}{(2)^{8-r}}\frac{(-4)^r}{x^r}$$

$$T_{r+1} = \binom{8}{r}\frac{(-4)^r}{(2)^{8-r}}x^{8-r-r}$$

$$T_{r+1} = \binom{8}{r}\frac{(-4)^r}{(2)^{8-r}}x^{8-2r} \rightarrow \text{(i)}$$

Put

$8 - 2r = 2$

$-2r = 2 - 8$

$-2r = -6$

$r = \frac{-6}{-2}$

$r = \boxed{3}$

Put $r = 3$ in eq.(i).

$$T_{3+1} = \binom{8}{3}\frac{(-4)^3}{(2)^{8-3}}x^{8-2(3)}$$

$$T_4 = 56\left(\frac{-64}{32}\right)x^{8-6}$$

$$T_4 = \boxed{-112x^2}$$

(v) $\left(\frac{p^2}{2}+6q^2\right)^{12}$: term involving q^8

(IIA-2018)

Sol. Here $n = 12$, $a = \frac{p^2}{2}$, $b = 6q^2$, $r = ?$

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SOLUTION OF EXERCISE # 2.1

$$T_{r+1} = \binom{n}{r} a^{n-r} b^r$$

$$T_{r+1} = \binom{12}{r}\left(\frac{p^2}{2}\right)^{12-r} (6q^2)^r$$

$$T_{r+1} = \binom{12}{r}\frac{(p)^{24-2r}}{(2)^{12-r}}(6)^r(q)^{2r} \text{ ______ (i)}$$

Put $2r = 8 \Rightarrow r = \frac{8}{2} = \boxed{4}$

Put $r = 4$ in eq.(i)

$$T_{4+1} = \binom{12}{4}\frac{(p)^{24-2(4)}}{(2)^{12-4}}(6)^4(q)^{2(4)}$$

$$T_5 = \binom{12}{4}\frac{(p)^{16}}{(2)^8}(6)^4(q)^8$$

$$T_5 = (495)\frac{p^{16}}{256}(1296)q^8$$

$$T_5 = \frac{641520}{256}p^{16}q^8 = \boxed{\frac{40095}{16}p^{16}q^8}$$

Q.8: Find the coefficients of:

(i) **x^5 in the expansion of** $\left(2x^2 - \frac{3}{x}\right)^{10}$

(IA-2016), (IIA-2021)

Sol. Here $n = 10$, $a = 2x^2$, $b = -\frac{3}{x}$, $r = ?$

$$T_{r+1} = \binom{n}{r} a^{n-r} b^r$$

$$T_{r+1} = \binom{10}{r}(2x^2)^{10-r}\left(-\frac{3}{x}\right)^r$$

SOLUTION OF EXERCISE # 2.1

$$T_{r+1} = \binom{10}{r}(2)^{10-r}x^{20-2r}\frac{(-3)^r}{x^r}$$

$$T_{r+1} = \binom{10}{r}(2)^{10-r}(-3)^r x^{20-2r-r}$$

$$T_{r+1} = \binom{10}{r}(2)^{10-r}(-3)^r x^{20-3r} \rightarrow (i)$$

Put

$20 - 3r = 5$

$-3r = 5 - 20$

$-3r = -15$

$r = \dfrac{-15}{-3}$

$\boxed{r = 5}$

Put $r = 5$ in eq.(i)

$$T_{5+1} = \binom{10}{5}(2)^{10-5}(-3)^5 x^{20-3(5)}$$

$$T_6 = 252(2)^5(-3)^5 x^{20-15}$$

$$T_6 = 252(32)(-243)x^5$$

$$T_6 = \boxed{-1959552x^5}$$

Hence, coefficient of x^5 is : $\boxed{-1959552}$

(ii) **x^{20} in the expansion of $\left(2x^2 + \dfrac{1}{2x}\right)^{16}$**

Sol. Here $n = 16$, $a = 2x^2$, $b = \dfrac{1}{2x}$, $r = ?$

$$T_{r+1} = \binom{n}{r}a^{n-r}b^r$$

$$T_{r+1} = \binom{16}{r}(2x^2)^{16-r}\left(\frac{1}{2x}\right)^r$$

$$T_{r+1} = \binom{16}{r}(2)^{16-r}(x^2)^{16-r}\frac{(1)^r}{(2)^r(x)^r}$$

$$T_{r+1} = \binom{16}{r}(2)^{16-r-r}\ x^{32-2r-r}$$

SOLUTION OF EXERCISE # 2.1

$$T_{r+1} = \binom{16}{r}(2)^{16-2r}\ x^{32-3r} \qquad \text{______ (i)}$$

Put

$32 - 3r = 20$

$-3r = 20 - 32$

$-3r = -12$

$r = \dfrac{-12}{-3}$

$r = \boxed{4}$

Put $r = 4$ in eq.(i)

$T_{4+1} = \binom{16}{4}(2)^{16-2(4)}\ x^{32-3(4)}$

$T_5 = \binom{16}{4}(2)^8 x^{20}$

$T_5 = (1820)(256)x^{20}$

$T_5 = 465920x^{20}$

Hence, coefficient of x^{20} is : $\boxed{465920}$

(iii) x^5 in the expansion of $\left(2x^2 - \dfrac{1}{3x}\right)^{10}$

Sol. Here $n = 10$, $a = 2x^2$, $b = \dfrac{-1}{3x}$, $r = ?$

$$T_{r+1} = \binom{n}{r}a^{n-r}b^r$$

$$T_{r+1} = \binom{10}{r}(2x^2)^{10-r}\left(-\frac{1}{3x}\right)^r$$

$$T_{r+1} = \binom{10}{r}(2)^{10-r}\ x^{20-2r}\ \frac{(-1)^r}{(3)^r x^r}$$

$$T_{r+1} = \binom{10}{r}\frac{(2)^{10-r}}{(3)^r}(-1)^r\ x^{20-2r-r}$$

$$T_{r+1} = \binom{10}{r}\frac{(2)^{10-r}}{(3)^r}(-1)^r\ x^{20-3r} \rightarrow (i)$$

SOLUTION OF EXERCISE # 2.1

Put

$20-3r=5$

$-3r=5-20$

$-3r=-15$

$r=\frac{-15}{-3}$

$r=\boxed{5}$

Put $r=5$ in eq.(i)

$T_{5+1}=\binom{10}{5}\frac{(2)^{10-5}}{(3)^5}(-1)^5x^{20-3(5)}$

$T_6=-\frac{(252)(32)(-1)}{243}x^5$

$T_6=-\frac{8064}{243}x^5=-\frac{896}{27}x^5$

Hence, coefficient of x^5 is : $\boxed{-\frac{896}{27}}$

(iv) **b^6 in the expansion of $\left(\frac{a^2}{2}+2b^2\right)^{10}$**

Sol. Here $n=10$, $a=\frac{a^2}{2}$, $b=2b^2$, $r=?$

$$T_{r+1}=\binom{n}{r}a^{n-r}b^r$$

$$T_{r+1}=\binom{10}{r}\left(\frac{a^2}{2}\right)^{10-r}(2b^2)^r$$

$$T_{r+1}=\binom{10}{r}\frac{a^{20-2r}}{(2)^{10-r}}(2)^r b^{2r}\rightarrow(i)$$

Put $2r=6 \Rightarrow r=6/2=\boxed{3}$

Put $r=3$ in eq.(i)

$$T_{3+1}=\binom{10}{3}\left(\frac{a^{20-2(3)}}{(2)^{10-3}}\right)(2)^3 b^{2(3)}$$

$$T_4=\frac{120a^{14}(8)b^6}{128}=\frac{960}{128}a^{14}b^6=\frac{15}{2}a^{14}b^6$$

Hence, coefficient of b^6 is : $\boxed{\frac{15}{2}a^{14}}$

SOLUTION OF EXERCISE # 2.1

Q.9: Find the constant term in the expansion of:

(i) $\left(x^2-\frac{1}{x}\right)^9$ (IA-2018), (IIA-2021), (IA-2022)

Sol. Here $n=9$, $a=x^2$, $b=-\frac{1}{x}$, $r=?$

$$T_{r+1}=\binom{n}{r}a^{n-r}b^r$$

$$T_{r+1}=\binom{9}{r}(x^2)^{9-r}\left(-\frac{1}{x}\right)^r=\binom{9}{r}x^{18-2r}\frac{(-1)^r}{x^r}$$

$$T_{r+1}=\binom{9}{r}(-1)^r x^{18-2r-r}$$

$$T_{r+1}=\binom{9}{r}(-1)^r x^{18-3r} \quad \to \text{(i)}$$

For constant term

Put $18-2r-r=0$

$-3r=-18$

$r=\frac{-18}{-3}$

$r=\boxed{6}$

Put $r=6$ in eq.(i)

$$T_{6+1}=\binom{9}{6}(-1)^6 x^{18-3(6)}$$

$T_7=84(1)x^0$

$T_7=84(1)(1)=\boxed{84}$

(ii) $\left(\sqrt{x}+\frac{1}{3x^2}\right)^{10}$ (IIA-2016)

Sol. Here $n=10$, $a=\sqrt{x}$, $b=\frac{1}{3x^2}$, $r=?$

$$T_{r+1}=\binom{n}{r}a^{n-r}b^r=\binom{10}{r}\left(\sqrt{x}\right)^{10-r}\left(\frac{1}{3x^2}\right)^r$$

$$T_{r+1}=\binom{10}{r}x^{5-\frac{r}{2}}\left(\frac{1}{3^r x^{2r}}\right)$$

SOLUTION OF EXERCISE # 2.1

$$T_{r+1} = \binom{10}{r}\frac{1}{3^r}x^{5-\frac{r}{2}-2r} \rightarrow (i)$$

For constant term

Put $5 - \frac{r}{2} - 2r = 0$

$$\frac{-r-4r}{2} = -5$$

$$-5r = -10$$

$$r = \frac{-10}{-5} = \boxed{2}$$

Put $r = 2$ in eq.(i)

$$T_{2+1} = \binom{10}{2}\frac{1}{(3)^2}x^0$$

$$T_3 = 45\left(\frac{1}{9}\right)(1)$$

$$T_3 = \boxed{5}$$

Q.10: Find the term independent of 'x' in the expansion of the following:

(i) $\left(2x^2 - \frac{1}{x}\right)^{12}$

Sol. Here $n = 12$, $a = 2x^2$, $b = -\frac{1}{x}$, $r = ?$

$$T_{r+1} = \binom{n}{r}a^{n-r}b^r$$

$$T_{r+1} = \binom{12}{r}(2x^2)^{12-r}\left(-\frac{1}{x}\right)^r = \binom{12}{r}(2)^{12-r}x^{24-2r}\frac{(-1)^r}{x^r}$$

$$T_{r+1} = \binom{12}{r}(2)^{12-r}(-1)^r x^{24-2r-r} \rightarrow (i)$$

For term independent of "x"

Put $24 - 2r - r = 0$

$$-3r = -24$$

$$r = \frac{-24}{-3} = \boxed{8}$$

Put $r = 8$ in eq.(i)

$$T_{8+1} = \binom{12}{8}(2)^{12-8}(-1)^8x^0$$

$$T_9 = 495(16)(1)(1)$$

$$T_9 = \boxed{7920}$$

SOLUTION OF EXERCISE # 2.1

(ii) $\left(2x^2+\frac{1}{x}\right)^9$ (IA-2017)

Sol. Here $n = 9$, $a = 2x^2$, $b = \frac{1}{x}$, $r = ?$

$$T_{r+1} = \binom{n}{r} a^{n-r} b^r$$

$$T_{r+1} = \binom{9}{r} (2x^2)^{9-r} \left(\frac{1}{x}\right)^r$$

$$T_{r+1} = \binom{9}{r} (2)^{9-r} x^{18-2r} \frac{1}{x^r}$$

$$T_{r+1} = \binom{9}{r} (2)^{9-r} x^{18-2r-r}$$

$$T_{r+1} = \binom{9}{r} (2)^{9-r} x^{18-3r} \rightarrow \text{(i)}$$

For term independent of " x "

Put $18 - 3r = 0$

$-3r = -18$

$r = \frac{-18}{-3} = 6$

Put $r = 6$ in eq.(i)

$$T_{6+1} = \binom{9}{6} (2)^{9-6} x^0$$

$T_7 = 84(8)(1)$

$T_7 = \boxed{672}$